7.2 Graph Exponential Decay Functions

Before You graphed and used exponential growth functions.
Now You will graph and use exponential decay functions.
Why? So you can model depreciation, as in Ex. 31.

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

Evaluate the expression without using a calculator.

1. $\left(\frac{1}{3}\right)^{-3}$ 27
2. $-\left(\frac{2}{5}\right)^{0}$ −1
3. $4\left(\frac{2}{3}\right)^{-2} + 1$ 10
4. A savings account pays 3% interest compounded monthly. What is the growth factor for this account? 1.0075

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 189–191.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with 7.1
• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1, p. 477

What does the graph of an exponential decay function look like?
Tell students they will learn how to answer this question by graphing a variety of exponential decay functions.

NCTM STANDARDS

Standard 8: Use the language of math to express ideas

Standard 10: Use representations to solve problems

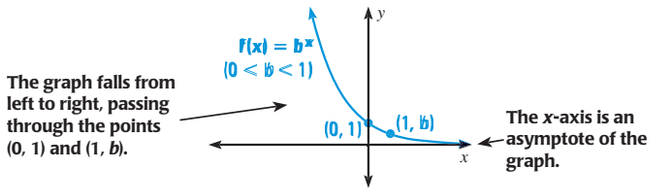
Key Vocabulary
- **exponential decay function**
- **decay factor**

In Lesson 7.1 you studied exponential growth functions. In this lesson, you will study **exponential decay functions**, which have the form $y = ab^x$ where $a > 0$ and $0 < b < 1$. The base b of an exponential decay function is called the **decay factor**.

KEY CONCEPT *For Your Notebook*

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 Graph $y = b^x$ for $0 < b < 1$

Graph $y = \left(\frac{1}{2}\right)^x$.

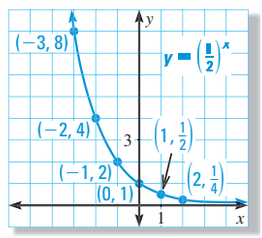
Solution

STEP 1 **Make** a table of values.

x	−3	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

STEP 2 **Plot** the points from the table.

STEP 3 **Draw**, from *right* to *left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 14–15)
- Activity Master (p. 16)
- Practice levels A, B, C (pp. 19–21)
- Study Guide (pp. 22–23)
- Catch-up for Absent Students (p. 24)
- Problem Solving Workshop (p. 25)
- Challenge (p. 26)

Workbooks
- Notetaking Guide (pp. 189–191)
- Practice Workbook (pp. 110–111)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the Algebra 2 Toolkit for more strategies for meeting individual needs.

TRANSFORMATIONS Recall from Lesson 7.1 that the graph of a function $y = ab^x$ is a vertical stretch or shrink of the graph of $y = b^x$, and the graph of $y = ab^{x-h} + k$ is a translation of the graph of $y = ab^x$.

EXAMPLE 2 Graph $y = ab^x$ for $0 < b < 1$

CLASSIFY FUNCTIONS
Note that the function in part (b) of Example 2 is not an exponential decay function because $a = -3 < 0$.

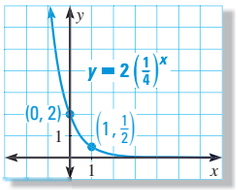
Graph the function.

a. $y = 2\left(\frac{1}{4}\right)^x$

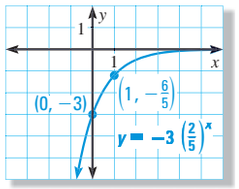
b. $y = -3\left(\frac{2}{5}\right)^x$

Solution

a. Plot (0, 2) and $\left(1, \frac{1}{2}\right)$. Then, from *right* to *left*, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the left.



b. Plot (0, −3) and $\left(1, -\frac{6}{5}\right)$. Then, from *right* to *left*, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the left.



Animated Algebra at classzone.com

GUIDED PRACTICE for Examples 1 and 2

Graph the function. 1–3. See margin.

1. $y = \left(\frac{2}{3}\right)^x$

2. $y = -2\left(\frac{3}{4}\right)^x$

3. $f(x) = 4\left(\frac{1}{5}\right)^x$

EXAMPLE 3 Graph $y = ab^{x-h} + k$ for $0 < b < 1$

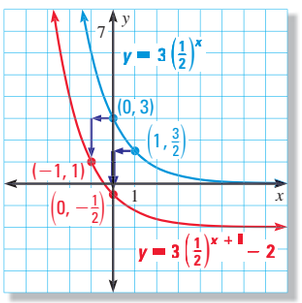
Graph $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$. State the domain and range.

Solution

Begin by sketching the graph of $y = 3\left(\frac{1}{2}\right)^x$, which passes through (0, 3) and $\left(1, \frac{3}{2}\right)$.

Then translate the graph left 1 unit and down 2 units. Notice that the translated graph passes through (−1, 1) and $\left(0, -\frac{1}{2}\right)$.

The graph's asymptote is the line $y = -2$. The domain is all real numbers, and the range is $y > -2$.



Differentiated Instruction

Auditory Learners Have students talk with each other about the transformations of the graph of the form $y = ab^x$ that occur in the function in **Example 3** where $b = \frac{1}{2}$. Have students make up their own functions of the form $y = ab^{x-h} + k$ for different a, h, and k, and then have them discuss the transformations with each other.

See also the Algebra 2 Toolkit for more strategies.

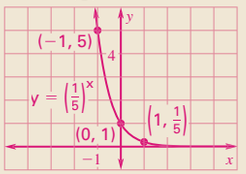
Motivating the Lesson

Your new computer cost $2600. The value of the computer decreases by about 15% each year. You can use an exponential decay function to estimate how much the computer will be worth in two years.

3 TEACH

Extra Example 1

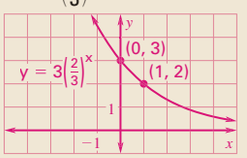
Graph $\left(\frac{1}{5}\right)^x$.



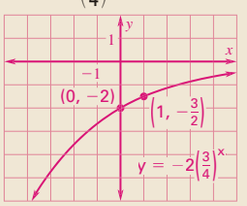
Extra Example 2

Graph the function.

a. $y = 3\left(\frac{2}{3}\right)^x$



b. $y = -2\left(\frac{3}{4}\right)^x$



Animated Algebra classzone.com

An **Animated Algebra** activity is available on-line for **Example 2**. This activity is also available on the **Power Presentations CD-ROM**.

1–3. See Additional Answers beginning on p. AA1.